Math 270: Differential Equations Day 12 Part 2

Section 4.6: Variation of Parameters Part 1

Section 4.6: Variation of Parameters (Part 1)

Variation of Parameters if another method of guessing a particular y_p solution to

$$ay'' + by' + cy = f(t)$$

Idea:

1) If y_1 and y_2 are 2 independent solutions to $ay'' + by' + cy = 0$, then we know all to $ay'' + by' + cy = 0$ is $y = Ay_1 + By_2$ where A and B are **constants**
2) We seek a particular solution to $ay'' + by' + cy = f(t)$ of the form $y_p = v_1y_1 + v_2y_2$ where v_1 and v_2 are **functions of *t***

3) How can we find v_1 and v_2 ?

Derive equations to find v_1 and v_2…to find 2 unknown functions, we need 2 equations

Section 4.6: Variation of Parameters (Part 1)

Derive equations to find v_1 and v_2…to find 2 unknown functions, we need 2 equations

Section 4.6: Variation of Parameters (Part 1)

Method of Variation of Parameters

To determine a particular solution to $ay'' + by' + cy = f$:

(a) Find two linearly independent solutions $\{y_1(t), y_2(t)\}$ to the corresponding homogeneous equation and take

$$y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t)\ .$$

(b) Determine $v_1(t)$ and $v_2(t)$ by solving the system in (9) for $v_1'(t)$ and $v_2'(t)$ and integrating.

$$\textbf{(9)} \qquad \begin{aligned} y_1v_1' + y_2v_2' &= 0 \\ y_1'v_1' + y_2'v_2' &= \frac{f}{a} \end{aligned}$$

(c) Substitute $v_1(t)$ and $v_2(t)$ into the expression for $y_p(t)$ to obtain a particular solution.

Section 4.6: Variation of Parameters (Part 1)

Example 1 Find a general solution on $(-\pi/2, \pi/2)$ to $\dfrac{d^2y}{dt^2} + y = \tan t$.

Section 4.6: Variation of Parameters (Part 1)

Example 2 Find a particular solution on $(-\pi/2, \pi/2)$ to $\dfrac{d^2y}{dt^2} + y = \tan t + 3t - 1$.